

# CROSS SECTION AND SINGLE TRANSVERSE TARGET SPIN ASYMMETRY FOR BACKWARD PION ELECTROPRODUCTION

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## Abstract

Nucleon to meson transition distribution amplitudes (TDAs), non-diagonal matrix elements of nonlocal three quark operators between a nucleon and a meson states, arise within the collinear factorized description of hard exclusive electroproduction of mesons off nucleons in the backward direction. Below we address the problem of modelling pion to nucleon TDAs. We suggest a factorized Ansatz for quadruple distributions with input from the soft pion theorem for  $\pi N$  TDAs. In order to satisfy the polynomiality property in its full form the spectral representation is complemented with a  $D$ -term like contribution from the nucleon exchange in the  $u$ -channel of the reaction. We present our estimates for the backward pion electroproduction unpolarized cross section and its transverse target single spin asymmetry within our composite model for  $\pi N$  TDAs.

The possibility to provide a description for hard exclusive electroproduction of mesons (specifically here pions) off nucleons

$$e(k_1) + N(p_1) \rightarrow (\gamma^*(q) + N(p_1)) + e(k_2) \rightarrow e(k_2) + \pi(p_\pi) + N'(p_2). \quad (1)$$

in terms of the fundamental degrees of freedom of QCD resides on the collinear factorization theorem [1] valid in the so-called generalized Bjorken limit: large  $Q^2 = -q^2$  and  $s = (p+q)^2$ ; fixed  $x_{\text{Bj}} = \frac{Q^2}{2(p \cdot q)}$  and the skewness  $\xi$ , defined with respect to the  $t$ -channel momentum transfer:  $\xi = -\frac{(p_2 - p_1) \cdot n}{(p_1 + p_2) \cdot n}$  ( $n$  is the conventional light cone vector occurring in the Sudakov decomposition of the relevant momenta) and small  $t$ -channel momentum transfer squared  $t \equiv (p_2 - p_1)^2$ . This factorization theorem allows to present the scattering amplitude as a convolution of the hard part (coefficient function - CF) with non-perturbative soft parts (generalized parton distributions - GPDs and distribution amplitudes - DAs) describing hadronic contents.

According to the conjecture made in [2], a similar collinear factorization theorem for the reaction (1) should be valid in the *complementary* kinematical regime: large  $Q^2$  and  $s$ ; fixed  $x_{\text{Bj}}$  and the skewness variable, which is now defined with respect to the  $u$ -channel momentum transfer  $\Delta \equiv p_\pi - p_1$ :  $\xi = -\frac{(\Delta \cdot n)}{(p_1 + p_\pi) \cdot n}$  and small  $u$ -channel momentum transfer squared  $u \equiv (p_\pi - p_1)^2$  (rather than small  $t$ ). Under these assumptions, referred to as the backward kinematics regime, the amplitude of the reaction (1) factorizes as it is presented on Fig.1 (see Ref. [5] for the detailed framework).

This requires the introduction of supplementary non-perturbative objects in addition to GPDs – nucleon to pion transition distribution amplitudes ( $\pi N$  TDAs) defined through the Fourier transform of the  $\pi N$  matrix element of the three-local quark operator on the light cone [3], [4]:

$$\hat{O}_{\rho\tau\chi}^{\alpha\beta\gamma}(\lambda_1 n, \lambda_2 n, \lambda_3 n) = \Psi_\rho^\alpha(\lambda_1 n) \Psi_\tau^\beta(\lambda_2 n) \Psi_\chi^\gamma(\lambda_3 n). \quad (2)$$

Here  $\alpha, \beta, \gamma$  stand for quark flavor indices and  $\rho, \tau, \chi$  denote the Dirac spinor indices; antisymmetrization in color is implied; gauge links are omitted in the light-like gauge  $A \cdot n = 0$ .

The physical picture encoded in baryon to meson TDAs is conceptually close to that contained in baryon GPDs [6]. By Fourier transforming baryon to meson TDAs to the impact parameter space ( $\Delta_T \rightarrow b_T$ ) a comprehensible three dimensional physical picture may be obtained. Baryon to meson TDAs encode complementary information on the hadron structure in the transverse plane. In particular, they allow to probe the localization of baryonic charge in the transverse plane and perform the femto-photography of hadrons [7] from a new perspective. There are also hints [8] that  $\pi N$  TDAs may be used as a tool to perform spatial imaging of the structure of nucleon's meson cloud. This point, which still awaits a detailed exploration, opens a fascinating window for the investigation of the various facets of the nucleon's interior.  $\pi N$  TDAs were recently estimated within the light cone quark model [9].

Below we briefly discuss how  $\pi N$  TDAs meet the fundamental requirements following from the symmetries of QCD, summarizing the main results of Refs. [10], [11].

- For given flavor contents spin decomposition of the leading twist-3  $\pi N$  TDA involve eight invariant functions  $V_{1,2}^{\pi N}, A_{1,2}^{\pi N}, T_{1,2,3,4}^{\pi N}$  each depending on the longitudinal momentum fractions  $x_i$  ( $\sum_{i=1}^3 x_i = 2\xi$ ), skewness parameter  $\xi$  and the  $u$ -channel momentum transfer squared  $\Delta^2 \equiv (p_\pi - p_1)^2$  as well as on the factorization scale  $\mu^2$ .
- Not all  $\pi N$  TDAs are independent. Taking the account of the isotopic and permutation symmetries (see [11]), one may check that in order to provide description of all isotopic channels of the reaction (1) it suffices to introduce eight independent  $\pi N$  TDAs: four in both the isospin- $\frac{1}{2}$  and the isospin- $\frac{3}{2}$  channels.
- The evolution properties of  $\pi N$  TDAs are described by the appropriate generalization [12] of the Efremov-Radyushkin-Brodsky-Lepage/ Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (ERBL/DGLAP) evolution equations.
- The support of  $\pi N$  TDAs in the longitudinal momentum fractions  $x_i$  is given by the intersection of three stripes  $-1 + \xi \leq x_i \leq 1 + \xi$  ( $\sum_{i=1}^3 x_i = 2\xi$ ) [10]. One can distinguish the ERBL-like domain, in which all  $x_i$  are positive and two type of DGLAP-like domains, in which one or two  $x_i$  turn negative.
- The polynomiality property for the Mellin moments of  $\pi N$  TDAs in the longitudinal momentum fractions  $x_i$  is the direct consequence of the underlying Lorentz symmetry. Similarly to the GPD case, the  $(n_1, n_2, n_3)$ -th ( $n_1 + n_2 + n_3 \equiv N$ ) Mellin moments of nucleon to meson TDAs in  $x_1, x_2, x_3$  are polynomials of power  $N$  or  $N + 1$  in the skewness variable  $\xi$ .

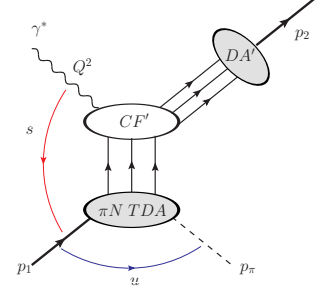


Figure 1: Factorization for hard production of pions off nucleons in the backward kinematics.

- Crossing transformation relates  $\pi N$  TDAs to  $\pi N$  generalized distribution amplitudes (GDAs), defined by the matrix element of the same operator (2) between the  $\pi N$  state and the vacuum. The soft pion theorem [13] for  $\pi N$  GDAs [14] also constrains  $\pi N$  TDAs near the soft pion threshold  $\xi = 1$ ,  $\Delta^2 = M^2$ , where  $M$  is the nucleon mass.

The most direct way to ensure both the polynomiality and the support properties for  $\pi N$  TDAs is to employ the spectral representation in terms of quadruple distributions [10]. Our strategy of modeling  $\pi N$  TDAs [15] is completely analogous to that employed for modeling nucleon GPDs with the help of Radyushkin's double distribution Ansatz [16]. The main difficulty is that, contrary to GPDs, baryon to meson TDAs lack a comprehensible forward limit ( $\xi = 0$ ). In order to propose a model for quadruple distributions it is illuminating to consider the alternative limit  $\xi = 1$ , in which  $\pi N$  TDAs are constrained by the chiral dynamics through the soft pion theorem [13] for  $\pi N$  GDAs. In this limit  $\pi N$  TDAs are expressed through the nucleon DAs  $\{V^p, A^p, T^p\}$  [17]. For example,  $\pi N$  TDAs  $V_1^{\pi^0 p}$ ,  $A_1^{\pi^0 p}$ ,  $T_1^{\pi^0 p}$  reduce to the following combination of the nucleon DAs [11]:

$$\begin{aligned} \{V_1^{\pi^0 p}, A_1^{\pi^0 p}\}(x_1, x_2, x_3, \xi = 1) &= -\frac{1}{4} \times \frac{1}{2} \{V^p, A^p\} \left( \frac{x_1}{2}, \frac{x_2}{2}, \frac{x_3}{2} \right); \\ T_1^{\pi^0 p}(x_1, x_2, x_3, \xi = 1) &= \frac{1}{4} \times \frac{3}{2} T^p \left( \frac{x_1}{2}, \frac{x_2}{2}, \frac{x_3}{2} \right), \end{aligned} \quad (3)$$

and  $\{V_2^{\pi^0 p}, A_2^{\pi^0 p}, T_2^{\pi^0 p}\}(x_1, x_2, x_3, \xi = 1) = -\frac{1}{2} \{V_1^{\pi^0 p}, A_1^{\pi^0 p}, T_1^{\pi^0 p}\}(x_1, x_2, x_3, \xi = 1)$ .

With appropriate change of spectral parameters the spectral representation for  $\pi N$  TDAs of Ref. [10] can be rewritten as:

$$\begin{aligned} H(w_i, v_i, \xi) &= \int_{-1}^1 d\kappa_i \int_{-\frac{1-\kappa_i}{2}}^{\frac{1-\kappa_i}{2}} d\theta_i \int_{-1}^1 d\mu_i \int_{-\frac{1-\mu_i}{2}}^{\frac{1-\mu_i}{2}} d\lambda_i \delta(w_i - \frac{\kappa_i - \mu_i}{2}(1 - \xi) - \kappa_i \xi) \\ &\times \delta(v_i - \frac{\theta_i - \lambda_i}{2}(1 - \xi) - \theta_i \xi) F(\kappa_i, \theta_i, \mu_i, \lambda_i). \end{aligned} \quad (4)$$

The index  $i = 1, 2, 3$  here refers to one of three possible choices of independent variables (quark-diquark coordinates):  $w_i = x_i - \xi$ ,  $v_i = \frac{1}{2} \sum_{k,l=1}^3 \varepsilon_{ikl} x_k$ . We suggest to use the following factorized Ansatz for the quadruple distribution  $F$  in (4):

$$F(\kappa_i, \theta_i, \mu_i, \lambda_i) = 4V(\kappa_i, \theta_i) h(\mu_i, \lambda_i), \quad (5)$$

where  $V(\kappa_i, \theta_i)$  is the combination of nucleon DAs to which  $\pi N$  TDA in question reduces in the limit  $\xi = 1$  (*c.f.* Eq. (3)), rewritten in terms of independent variables:  $\kappa_i = 2y_i - 1$ ;  $\theta_i = \sum_{k,l=1}^3 \varepsilon_{ikl} y_k$ .

The profile function  $h(\mu_i, \lambda_i)$  is normalized as  $\int_{-1}^1 d\mu_i \int_{-\frac{1-\mu_i}{2}}^{\frac{1-\mu_i}{2}} d\lambda_i h(\mu_i, \lambda_i) = 1$ . The support of the profile function  $h$  is also that of a baryon DA. The simplest assumption for the profile is to take it to be determined by the asymptotic form of baryon DA ( $120y_1y_2y_3$  with  $\sum_{i=1}^3 y_i = 1$ ) rewritten in terms of variables  $\mu_i = 2y_i - 1$ ,  $\lambda_i = \sum_{k,l=1}^3 \varepsilon_{ikl} y_k$ :

$$h(\mu_i, \lambda_i) = \frac{15}{16} (1 + \mu_i)((1 - \mu_i)^2 - 4\lambda_i^2). \quad (6)$$

Similarly to the GPD case [18], in order to satisfy the polynomiality condition in its complete form the spectral representation for  $\pi N$  TDAs  $\{V_{1,2}, A_{1,2}, T_{1,2}\}^{\pi N}$  should be complemented by a  $D$ -term like contribution. The simplest possible model for such a  $D$ -term is the contribution of the  $u$ -channel nucleon exchange into  $\pi N$  TDAs computed in [11]. In this way we come to a two component model for  $\pi N$  TDAs involving the spectral representation part, based on the factorized Ansatz (5) with the profile (6) and with input from the soft pion theorem, and the  $D$ -term, originating from the nucleon exchange in the  $u$ -channel. It provides a model for  $\pi N$  TDAs in the complete domain of their definition.

Within the factorized approach the leading order (both in  $\alpha_s$  and  $1/Q$ ) amplitude of backward hard pion production  $\mathcal{M}_{s_1 s_2}^\lambda$  reads [5]:

$$\mathcal{M}_{s_1 s_2}^\lambda = \mathcal{C} \frac{1}{Q^4} \left[ \mathcal{S}_{s_1 s_2}^\lambda \mathcal{I}(\xi, \Delta^2) + \mathcal{S}_{s_1 s_2}'^\lambda \mathcal{I}'(\xi, \Delta^2) \right]. \quad (7)$$

The spin structures  $\mathcal{S}$  and  $\mathcal{S}'$  are defined as  $\mathcal{S}_{s_1 s_2}^\lambda \equiv \bar{U}(p_2, s_2) \hat{\mathcal{E}}(\lambda) \gamma^5 U(p_1, s_1)$ ;  $\mathcal{S}_{s_1 s_2}'^\lambda \equiv \frac{1}{M} \bar{U}(p_2, s_2) \hat{\mathcal{E}}(\lambda) \hat{\Delta}_T \gamma^5 U(p_1, s_1)$ , where  $\mathcal{E}$  denotes the polarization vector of the virtual photon and  $U$  is the usual nucleon Dirac spinor.  $\mathcal{C}$  is the normalization constant  $\mathcal{C} \equiv -i \frac{(4\pi\alpha_s)^2 \sqrt{4\pi\alpha_{em}} f_N^2}{54 f_\pi}$ , where  $\alpha_{em}(\alpha_s)$  stands for the electromagnetic (strong) coupling,  $f_\pi = 93\text{MeV}$  is the pion decay constant and  $f_N$  is the normalization constant of the nucleon DA [17].

The coefficients  $\mathcal{I}, \mathcal{I}'$  result from the calculation of 21 diagrams contributing to the hard scattering amplitude [5]:

$$\{\mathcal{I}, \mathcal{I}'\}(\xi, \Delta^2) \equiv \int d^3x \delta(\sum_i x_i - 2\xi) \int d^3y \delta(\sum_i y_i - 1) \left( 2 \sum_{\alpha=1}^7 \{T_\alpha, T'_\alpha\} + \sum_{\alpha=8}^{14} \{T_\alpha, T'_\alpha\} \right),$$

where the convolution integrals in  $x_i$  and  $y_i$  stand over the supports of  $\pi N$  TDAs and nucleon DAs respectively. The explicit expressions for the coefficients  $T_\alpha$  and  $T'_\alpha$  for  $\gamma^* p \rightarrow \pi^0 p$  channel are presented in the Table I of Ref. [5]. The result for  $\gamma^* p \rightarrow \pi^+ n$  channel can be read off the same Table with the obvious changes:  $Q^u \rightleftharpoons Q^d$ ;  $\{V_{1,2}, A_{1,2}, T_{1,2,3,4}\}^{p\pi^0} \rightarrow \{V_{1,2}, A_{1,2}, T_{1,2,3,4}\}^{p\pi^+}$ ;  $\{V, A, T\}^p \rightarrow \{V, A, T\}^n$ . In [15] we develop a reliable method for the calculation of the corresponding convolution integrals.

Within the suggested factorization mechanism for backward pion electroproduction only the transverse cross section  $\frac{d^2\sigma_T}{d\Omega_\pi}$  receives a contribution at the leading twist level. We establish the following formula for the unpolarized transverse cross section through the coefficients  $\mathcal{I}, \mathcal{I}'$  introduced in (7):

$$\frac{d^2\sigma_T}{d\Omega_\pi} = |\mathcal{C}|^2 \frac{1}{Q^6} \frac{\Lambda(s, m^2, M^2)}{128\pi^2 s(s - M^2)} \frac{1 + \xi}{\xi} (|\mathcal{I}|^2 - \frac{\Delta_T^2}{M^2} |\mathcal{I}'|^2), \quad (8)$$

where  $\Lambda(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2xy - 2xz - 2yz}$  is the usual Mandelstam function. Within our two component model for  $\pi N$  TDAs  $\mathcal{I}$  receives contributions both from the spectral representation component and nucleon pole exchange contribution.  $\mathcal{I}'$  is determined solely by the nucleon pole contribution. On Fig. 2 we present our estimates for the unpolarized cross section  $\frac{d^2\sigma_T}{d\Omega_\pi}$  of backward production of  $\pi^+$  and  $\pi^0$  off protons for  $Q^2 = 10\text{GeV}^2$  and  $u = -0.5\text{GeV}^2$  in nb/sr. CZ solution [17] for the nucleon DAs is used

as phenomenological input for our model. The magnitude of the cross sections is large enough for a detailed investigation to be carried at high luminosity experiments such as J-lab@12GeV and EIC. The scaling law for the unpolarized cross section (8) is  $1/Q^8$ .

Asymmetries, being ratios of the cross sections, are less sensitive to perturbative corrections. Therefore, they are usually considered as more reliable observables to test the factorized description of hard reactions. For the backward pion electroproduction the evident candidate is the single transverse target spin asymmetry (STSA) [19] defined as:

$$\begin{aligned} \mathcal{A} &= \frac{1}{|\vec{s}_1|} \frac{\left( \int_0^\pi d\tilde{\varphi} |\mathcal{M}_T^{s_1}|^2 - \int_\pi^{2\pi} d\tilde{\varphi} |\mathcal{M}_T^{s_1}|^2 \right)}{\left( \int_0^{2\pi} d\tilde{\varphi} |\mathcal{M}_T^{s_1}|^2 \right)} \\ &= -\frac{4}{\pi} \frac{\frac{\Delta_T}{M} \text{Im}(\mathcal{I}'(\mathcal{I})^*)}{|\mathcal{I}|^2 - \frac{\Delta_T^2}{M^2} |\mathcal{I}'|^2}. \end{aligned} \quad (9)$$

Here  $\tilde{\varphi} \equiv \varphi - \varphi_s$ , where  $\varphi$  is the angle between leptonic and hadronic planes and  $\varphi_s$  is the angle between the leptonic plane and the transverse spin of the target. Our two component model for  $\pi N$  TDAs provides a non-vanishing numerator in the last equality of (9) thanks to the interfering contributions of the spectral part into  $\text{Im}\mathcal{I}(\xi)$  and of the nucleon pole part into  $\text{Re}\mathcal{I}'(\xi)$ .

On Fig. 3 we show the result of our calculation of the STSA for backward  $\pi^+$  and  $\pi^0$  electroproduction off protons for  $Q^2 = 10 \text{ GeV}^2$  and  $u = -0.5 \text{ GeV}^2$ . STSA turns out to be sizable in the valence region and its measurement should be considered as a crucial test of the applicability of our collinear factorized scheme for backward pion electroproduction.

Our estimates of backward pion electroproduction cross section and single transverse spin asymmetry make us hope for bright experimental prospects for measuring baryon to meson TDAs with high luminosity electron beams such as J-lab@ 12 GeV and EIC. Experimental data from J-lab@ 6 GeV on backward  $\pi^+$ ,  $\pi^0$ ,  $\eta$  and  $\omega$  meson production are currently being analyzed [20]. We eagerly await for the experimental evidences for validity of the factorized picture of backward electroproduction reactions suggested in our approach.

This work is supported in part by the Polish NCN grant DEC-2011/01/B/ST2/03915 and by the French-Polish Collaboration Agreement Polonium.

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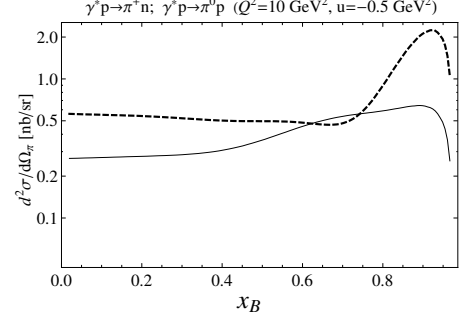


Figure 2: Unpolarized cross section for backward  $\pi^+$  (solid) and  $\pi^0$  (dashed) production off proton.

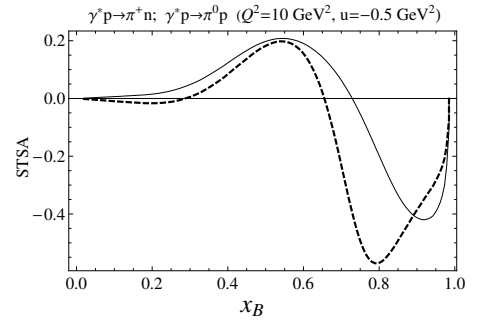


Figure 3: Single transverse spin asymmetry for backward  $\pi^+$  (solid) and  $\pi^0$  (dashed) production off proton.

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